From Hebbian learning to spike-timing-dependent plasticity A modeling viewpoint

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## Outline

- Historical perspective on learning in neuronal systems
- From Hebbian learning to spike-timing-dependent plasticity
- Weight dynamics and "information" representations:
  - Principal component analysis (PCA)
  - Detection of spike patterns by STDP
  - Neuronal assemblies in recurrent networks
- From biological learning to machine learning:
  - Unsupervised vs supervised vs reinforcement learning

## A bit of history

- Paul Broca (1796-1881): localization of function, for example language
- Karl Lashley (1890-1958): storage of memory in brain regions (engram)
- How are brain functions (psychology) implemented in the brain?
- How to learn functions?



Ramon y Cajal 1905

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Ramon y Cajal 1905

Hebb (1949): "When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased"





motor cortex



## Formalizing experimental observations



Helmstaedter BRR 2007

## Formalizing experimental observations







Tuning function e.g. pattern classification







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## Spike timing matters!



- Markram Science 1997
- Gerstner *Science* 1999

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## Spike timing matters!



Markram J Syn Neurosci 2011

Spike-timing-dependent plasticity (STDP): temporally Hebbian, "takes part in firing it"

### Many STDP windows exist, yielding various timescales





## Biophysical models and other mechanisms

- Weight dependence: van Rossum J Neurosci 2000, Morrison Neural Comput 2007
- Calcium-based: Shouval Biol Cybern 2002; Standage PLoS ONE 2014
- Post-synaptic voltage dependence: Clopath Nat Neurosci 2010
- Inhibitory plasticity: Vogels Science 2011
- Neuronal morphology: Froemke Nature 2005
- Neuromodulation: Brzosko Neuron 2019



## Summary for experimental evidence and modeling of synaptic plasticity

- Complex dependencies upon recent spike timing, ion concentrations, etc.
- But most experiments are in vitro! (see supplementary slides)
  - Many STDP protocols involve 60 repeated pairing to obtain observable weight change
  - Positive replications of results in-vivo still scarce...
- Monosynaptic plasticity is not the only mechanism at play

# Summary for experimental evidence and modeling of synaptic plasticity

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#### • Questions:

- What matters for neuronal function?
- Effect in network?
- Need to formalize plasticity update (i.e. simplified model) so we can build typology of functional effects

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## Stimulus represented in rate patterns



stimulus orientation

Orientation selectivity of V1 neuron

Orientation selectivity of its inputs (LGN)

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## Stimulus represented in rate patterns



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Orientation selectivity of V1 neuron

Orientation selectivity of its inputs (LGN)



## On abstract level: rate-pattern recognition

most frequently repeated pattern



less frequently repeated pattern



## Weight dynamics for rate neuron



 $\dot{w}_i \propto x_i y$


$$\dot{w}_i \propto x_i y = \sum_j x_i x_j w_j$$



$$\dot{w}_i \propto x_i y = \sum_j x_i x_j w_j$$

$$\dot{w} = C w$$
 with  $C_{ij} = \langle x_i x_j \rangle$ 



$$\dot{w}_i \propto x_i y = \sum_j x_i x_j w_j$$

Hebbian dynamics is intrinsically unstable!

Main direction  $w \sim e^{\lambda_{r}}$ 

$$\dot{w} = C w$$
 with  $C_{ij} = \langle x_i x_j \rangle$ 

$$v \sim e^{\lambda_{max}t} [V_{max}.w(t=0)] V_{max}$$

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$$\dot{w}_i \propto x_i y = \sum_j x_i x_j w_j$$

Hebbian dynamics is intrinsically unstable!

> Main direction of growth

$$\dot{w} = C w \quad \text{with} \quad C_{ij} = \langle x_i x_j \rangle$$
$$w \sim e^{\lambda_{max} t} [V_{max} \cdot w(t=0)] V_{max}$$

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max

## Stabilization by synaptic competition: Oja's rule

$$\dot{w}_i = y(x_i - yw_i)$$

Oja J Math Biol 1982

## Stabilization by synaptic competition: Oja's rule

$$\dot{w}_i = y(x_i - yw_i)$$
stabilization effect  $\sum_j w_j \sim const$ 

Oja J Math Biol 1982

#### Stabilization by synaptic competition: Oja's rule

$$\dot{w}_i = y(x_i - yw_i)$$

$$\sum_{j} w_{j} \sim const$$

- Select dominating eigenvector of C
- Principal component analysis (PCA)



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#### Unsupervised learning for detection of spike patterns



$$\dot{w}_{\dots} = F(t_{pre} - t_{post}) - a_{pre}$$

Masquelier PLoS ONE 2008

#### Unsupervised learning for detection of spike patterns



output neuron becomes selective (LTP for synapses from early spikes)



$$\dot{w}_{\dots} = F(t_{pre} - t_{post}) - a_{pre}$$

Masquelier PLoS ONE 2008

### Learning the full pattern, not just the start



Masquelier Neural Comput 2009

#### Maths for weight dynamics





$$\rho(t) = \sum_{j} w_{j} [\epsilon * s_{j}](t)$$

$$\dot{w}_i = \sum_{t_i, t_{post}} F(t_i - t_{post})$$



$$\rho(t) = \sum_{j} w_{j} [\epsilon * s_{j}](t)$$
$$P[u(t)=1] \propto \rho(t)$$

$$\dot{w}_{i} = \sum_{t_{i}, t_{post}} F(t_{i} - t_{post})$$
$$= \int_{t} \int_{\tau} F(\tau) s_{i}(t) u(t - \tau) d\tau dt$$



 $\rho(t) = \sum_{j} w_{j} [\epsilon * s_{j}](t)$  $P[u(t) = 1] \propto \rho(t)$ 

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=  $\sum_{j} w_{j} \int_{\tau} [F * \epsilon](\tau) C_{ij}(\tau) d\tau$ 

spike-time correlogram

$$C_{ij}(\tau) = \langle s_i(t) s_j(t-\tau) \rangle$$



 $\rho(t) = \sum_{j} w_{j} [\epsilon * s_{j}](t)$  $P[u(t) = 1] \propto \rho(t)$ 

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$$= \sum_{j} w_{j} \int_{\tau} [F * \epsilon](\tau) C_{ij}(\tau) d\tau$$

$$\dot{w} = \tilde{C} w$$

spike-time correlogram

$$C_{ij}(\tau) = \langle s_i(t) s_j(t-\tau) \rangle$$

#### STDP selects early, dense and sharp spike clusters

competition between spike clusters (PCA on correlation structure)



PCA becomes ICA with additional regularization:

$$\dot{w} = \tilde{C} w - a_{pre}$$

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## Weight dynamics in recurrent networks



$$\dot{w}_{ij} = \int_{\tau} F(\tau) \langle s_j(t) s_i(t-\tau) \rangle d\tau$$
$$\rho_i(t) = \sum_j w_{ij} [\epsilon * s_j](t) + \sum_k w_{ik} [\epsilon * s_k](t)$$

But  $\boldsymbol{s}_i$  depends on  $\boldsymbol{s}_i$  too!

Echos in networks due to recurrent connections

Gilson Biol Cybern 2009





$$\dot{w}_{ij} = \int_{\tau} F(\tau) C(\tau) d\tau$$

Gilson Biol Cybern 2009









Gilson Biol Cybern 2009

#### Stability in recurrent networks with ongoing plasticity



Zenke Nat Comm 2015

#### Stability in recurrent networks with ongoing plasticity



$$\dot{w} = F(\lbrace t_{pre} \rbrace, \lbrace t_{post} \rbrace) - \beta(w - w^{eq})r_{post}^4 + \alpha r_{pre}$$

Zenke Nat Comm 2015

# Summary for plastic weight dynamics

- Learned weight structure represent the input statistics and shape the neuronal function (input-output mapping)
  - implement selectivity to pattern
  - create cell assemblies that receive correlated inputs
- Beware of weight instability!
  - other mechanisms are necessary to stabilize learning, like heterosynaptic plasticity that models resource limitation

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# Learning in neuronal systems



# Types of learning

Ŵ unsupervised (Hebbian, STDP)

$$w_i = y x_i$$

supervised (delta rule, perceptron)

$$\dot{w}_i = (\overline{y}^A - y) x_i^A \qquad \overline{y}^A$$
 objective for input of class A

# Types of learning

unsupervised  $\dot{w}_i = j$ (Hebbian, STDP)

$$\dot{v}_i = y x_i$$

supervised (delta rule, perceptron)

$$\dot{w}_i = (\bar{y}^A - y) x_i^A$$

 $\overline{y}^{A}$  objective for input of class A

reinforcement learning

$$\dot{w}_i = \epsilon y x_i$$

€ modulator (dopamine, acetylcholine)

#### **Reward-modulated STDP**



Izhikevich Cereb Cortex 2007

#### **Reward-modulated STDP**



Izhikevich Cereb Cortex 2007
#### Presentation available on http://www.matthieugilson.eu















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# Supplementary material about STDP

- Experimental evidence of STDP: how rich should the model be?
- Weight dynamics and "information" representations:
  - Ocular dominance and symmetry breaking
- Future challenges:
  - Distributed information representations

# STDP: what do experiments really say?



#### Weight change after 60 pre-post pairings!



from review Feldman *Neuron* (2012) data: Bi and Poo (1998), Feldman (2000) stochastic STDP model: Elliott (2008)

## Beyond spike pairs



data: Sjostrom *Neuron* (2001) model: Pfister *J Neurosci* (2006)

# Beyond spike pairs



 $\Delta t_1 \Delta t_2$ 

Experiments All-to-All

Nearest-Spike

(10,-10)

 $(\Delta t_1, \Delta t_2)$  [ms]

(15,-5)

(5.-15)

C

<sup>∞</sup>0.1

0.2

0.2

-0.05

-0.

(5,-5)



STDP based on triplet of spikes in addition to spike pairs



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### Ocular dominance

- Symmetric inputs from the two eyes
- How to specialize to one optical input only?



### Ocular dominance

- Symmetric inputs from the two eyes
- How to specialize to one optical input only?
- PCA cannot discriminate between the 2 dominating eigenvectors



#### BCM rule

- Symmetric inputs from the two eyes
- How to specialize to one optical input only?
- PCA cannot discriminate between the 2 dominating eigenvectors



Bienenstock, Cooper, Munro J Neurosci 1981

# Triplet STDP and BCM



enforces winner-take-all behavior: strong specialization and symmetry breaking

Pfister J Neurosci 2006

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# Towards theory for distributed computations



Information representation Input-output mapping

Learning

Synaptic

plasticity

# Neuronal information in high-order correlations



**B** Classification based on mean mapping (perceptron)



**D** matrices

Classification based on covariance mapping



Gilson M, D Dahmen, Moreno-Bote R, Insabato A, Helias M *PLoS Comput Biol* 2020



Gilson M, Pfister J-P (*arXiv*) 85

# Further reading on link with information theory

PLoS Comput Biol. 2013 Apr;9(4):e1003037. doi: 10.1371/journal.pub. 1000001. Lpub 2010 Apr;20.

# Bayesian computation emerges in generic cortical microcircuits through spike-timing-dependent plasticity.

Nessler B<sup>1</sup>, Pfeiffer M, Buesing L, Maass W.

<u>Neural Comput.</u> 2016 Sep;28(9):1859-88. doi: 10.1162/NECO\_a\_00862. Epub 2016 Jul 8.

# Linking Neuromodulated Spike-Timing Dependent Plasticity with the Free-Energy Principle.

Isomura T<sup>1</sup>, Sakai K<sup>2</sup>, Kotani K<sup>3</sup>, Jimbo Y<sup>4</sup>.

#### Eligibility Traces and Plasticity on Behavioral Time Scales: Experimental Support of NeoHebbian Three-Factor Learning Rules.

<u>Gerstner W</u><sup>1</sup>, Lehmann M<sup>1</sup>, Liakoni V<sup>1</sup>, Corneil D<sup>1</sup>, Brea J<sup>1</sup>.





