

timation of effective dynamics and network analysis from whole-brain fMRI data

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Effective connectivity ~ signature of brain dynamics (related to transitions of BOLD activity between ROIs)

Matrix of effective connectivity:

• Changes across conditions



www.biorxiv.org/content/early/2017/08/25/110015

Matrix of effective connectivity:

- Changes across conditions
- Important and strong links



Weight significantly different from 0

~ Granger causality analysis

- Changes across conditions
- Important and strong links
- Network effect





- Changes across conditions
- Important and strong links
- Network effect



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Matrix of effective connectivity:

- Changes across conditions
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How to make sense of the multiple superposed paths for the network dynamics?





Outline

- Non-parametric method to evaluate significance of estimated weights
 - multivariate autoregressive process (discrete time)
 - www.biorxiv.org/content/early/2017/06/26/100669
- Communicability to quantify network effect
 - multivariate Ornstein-Uhlenbeck process (continuous time)

How to detect significantly strong interactions?

- Estimated connectivity from multiunit activity from UTAH electrode array (in monkey V1) from Alex Thiele (U Newcastle)
- Visual stimulus in a bottom-up attention task





time relative to trial start

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Theoretical study with synthetic networks



Generative model: multivariate autoregressive model (linear feedback with discrete time)

$$x^t = Ax^{t-1} + \zeta^t \; ,$$

Parametric testing for Granger causality analysis

Autoregression from observed time series

$$x_i^t = \sum_{j \in \mathcal{S}} A_{ij} x_j^{t-1} + \epsilon^t$$

Residual noise

$$\epsilon \left(x_i^{2 \le t \le T} | x_i^{1 \le t \le T-1} \right) = \sqrt{\sum_t (\epsilon^t)^2}$$

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Log ratio

$$\operatorname{GRu}(x_j \to x_i) = \ln \left[\frac{\epsilon \left(x_i^{2 \le t \le T} | x_i^{1 \le t \le T-1} \right)}{\epsilon \left(x_i^{2 \le t \le T} | x_{i,j}^{1 \le t \le T-1} \right)} \right]$$

F test for significance (T samples)

$$\left[\exp(\mathrm{GRu}_{ij}) - 1\right] > \frac{\phi(\alpha, 1, T - 3)}{T - 3}$$

Is x_j useful to predict x_i?



,

Non-parametric test for estimated MVAR coefs?



Maximum likelihood estimate obtained from covariances (simpler than continuous time)

$$A = \widehat{Q}^1 (\widehat{Q}^0)^{-1}$$

Shuffling the observed time series

- Destroys covariance structure to build surrogate
- Distribution of estimated MVAR coefficients for absent connections (blue) is matched by surrogate distribution (black)





$$A = \widehat{Q}^1 (\widehat{Q}^0)^{-1}$$

- Local: each connection has its surrogate distribution
- Global: pooling all surrogate matrix elements



local/global significance threshold (p=0.01 \leftrightarrow 1%-tail of the distribution)

- Local: each connection has its surrogate distribution
- Global: pooling all surrogate matrix elements



false alarm: wrong detection of absent connection

network with N=100 nodes and 5-30% density: 1% false alarms corresponds to 70-95 connections!



- Local: each connection has its surrogate distribution
- Global: pooling all surrogate matrix elements



- false alarm: wrong detection of absent connection
- **miss:** fail to detect existing connection



50 versus 400 surrogates

- Local: each connection has its surrogate distribution
- Global: pooling all surrogate matrix elements



With local test, more surrogates (darker red) reduce the miss rate (especially for small weights, left violin plots)



Comparison with Granger causality analysis

- Local test (red) and global test (gray)
- Comparison with conditional Granger causality analysis:
 - parametric test (reference in B, zero on y-axis)
 - non-parametric test (green)



- Local test always better for S>100 surrogates
- Conditional Granger causality analysis doesn't work well in network with redundant information (Stramaglia et al. 2014)
 - high feedback due to density

For the considered dense networks, estimated MVAR coefficients are better aligned with the original weights (Spearman correlation)

- Blue: unconditional Granger residuals
- Cyan: conditional Granger residuals
- Red: MVAR coefficients



Summary for detection of significant connections

- Surrogates generated by shuffling observed time series can be used to perform statistical testing for MVAR estimates
- Robust to network topology, works with 2nd-order MVAR
- Computational cost is reasonable

- Future work:
- Also: better characterize what information is extracted from MUAe (monkey data)



www.biorxiv.org/content/early/2017/06/26/100669 (accepted in Network Neuroscience) How to make sense of the multiple superposed paths for the network dynamics?





Communicability for graphs (structure→function)

- Measures indirect interactions
- Involves both shortest paths and longer paths (Estrada and Hatano 2008)



number of paths between i and k of lengths 1, 2, 3, etc. from adjacency matrix A:

$$A_{ik}, (A^2)_{ik}, (A^3)_{ik}, \dots$$

Communicability for graphs (structure→function)

- Measures indirect interactions
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decay for longer paths

$$e^{A} = 1 + \frac{A}{1!} + \frac{A^{2}}{2!} + \frac{A^{3}}{3!} + \dots$$

from i to k:



Communicability for static networks

- Initially used to detect communities in undirected graphs
 - communicability for pairs of nodes as a function of their degrees (k_p, k_q)



Estrada and Hatano 2008

Communicability for static networks

- Initially used to detect communities in undirected graphs
 - communicability for pairs of nodes as a function of their degrees (k_p, k_q)
- Also used to relate brain structure to function
 - anatomical (DTI) → functional (fMRI) connectivity

Bettinardi, Deco, Zamora-López et al., Chaos (2017)



DTI





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DTI





7.5

Formal relationship with noise-diffusion model

multivariate Ornstein-Uhlenbeck process

$$dx = \left(\frac{-x}{\tau} + Cx\right)dt + dW$$

- Effective connectivity matrix C (transition matrix)
- Decay time constant $\boldsymbol{\tau}$
- Wiener process W (white noise)

Formal relationship with noise-diffusion model

multivariate Ornstein-Uhlenbeck process

$$dx = \left(\frac{-x}{\tau} + Cx\right) dt + dW \qquad \longrightarrow \qquad J_{ij} = \frac{-\delta_{ij}}{\tau} + C_{ij}$$

- Effective connectivity matrix C (transition matrix)
- Decay time constant $\boldsymbol{\tau}$
- Wiener process W (white noise)
- Jacobian J = propagator of dynamics

Forward equation to predict future state:

$$x(t) = e^{Jt} x(0) + \int_0^t e^{J(t-s)} dW(s)$$

Formal relationship with noise-diffusion model

multivariate Ornstein-Uhlenbeck process





- EC ~ transition matrix
- Matrix exponential is Green function of network dynamics
- Estrada and Hatano (2008): static viewpoint (t=1)

Dynamic communicability for noise-diffusion network

Comparison to null model with no connectivity (only local dynamics)

$$K^{t} = \frac{e^{Jt} - e^{J^{0}t}}{\|(J^{0})^{-1}\|}$$

$$J_{ij} = \frac{-\delta_{ij}}{\tau} + C_{ij}$$

Jacobian for null model

$$J_{ii}^{0} = \frac{-\delta_{ij}}{\tau}$$



Time-dependent analysis of communicability





Quantification of network feedback

• Open loop < closed loops



Quantification of network feedback

- Open loop < closed loops
- Dynamic communicability captures heterogeneities between nodes



Quantification of network feedback

- Open loop < closed loops
- Dynamic communicability captures heterogeneities between nodes
- Homogenization over time





Application to resting-state fMRI

- Data from JF Mangin (Neurospin)
- 57 subjects
- AAL90 parcellation
- TR = 2 seconds
- Strong network effect (long lasting)
 - close to criticality
- Input-output heterogeneities across nodes
 - e.g., precuneus listens but does not broadcast



From segregated to global integration

- Panel B: 4 communities detected from EC weights using the Louvain method (Newman modularity)
- Communicability initially increases within communities, then spreads (cf. off-diagonal blocks)
 - link with Laplacian-flow to detect communities (Rosvall and Bergstrom; Lambiotte, Bahorona)



Summary for dynamic communicability

- It quantifies network interactions across time
- Time-dependent measures (e.g., to compare distinct conditions)
 - global network effect
 - integration and broadcast strengths for each node
 - segregated \rightarrow global integration (community merging over time)
- Definition of flow to take the input statistics (variances) into account in the noise-diffusion process



Conclusions

- We aim to develop an analysis framework for complex network dynamics estimated from data (e.g., fMRI, MUAe)
 - extend graph theory for static network
 - beyond mean-field approaches for dynamic systems
 - adequate statistical tests
- Code for EC estimation available online (matthieugilson.eu)



Acknowledgments

Data:

- JF Mangin, D Rivière (Neurospin, Paris)
- A Thiele (U Newcastle)

Funding:

- ERC Advanced Grant
- Human Brain Project
- MSCA fellowship

